Common Sense Problem Solving and Cognitive Research

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To solve the typical textbook problem, an experienced person will scan the problem, make a mental marker that places the problem in the appropriate field of knowledge (for example, kinematics, electricity and magnetism, quantum mechanics, or some other field). Continuing the scan, the expert makes another mental marker for the next more precise area (perhaps Newton's second law, Faraday's law...).

As the scan goes on, there may be a number of such markers further refining the concepts needed to solve the problem. These markers are held in the expert's mind, usually without that person being consciously aware of what's happening. The trained mind is very powerful.

When the specific parameters of the problem are reached, the expert begins making notations and works back up through the hierarchy which has been marked out during the scanning. This results in presenting the solution in reverse order from the actual solution that was done. The examples in textbooks are therefore very natural presentations from the expert's point of view.

The inverted presentation is confusing to the novice who has indeed had considerable training in non-inverted processes. Since problem solving is a linear process, even though the ideas that evolve in the process of solving a problem are far from linear, it is necessary to present problem solutions linearly. Doing so involves the use of linguistics and semantics.

The student, however, does not yet have the ability or resources to make these mental markers and carry them along in the mind. The student needs to see these guideposts written in the order in which they are encountered. Each new branch that narrows the area of investigation is actually a subproblem of the broader level concept and needs to be recognized as such.

The problem solution presentations included here are a sample of solutions designed to provide students with problem solving that is more representative of the process an expert actually goes through than the abbreviated, inverted presentations shown in textbooks and solution manuals. These are a small part of a substantial database of problem solutions currently being edited into publishable form.

The references listed below are not complete but serve to indicate the dimensions of the research and experience that are the basis for the manner in which these solutions are presented. The process of solving problems is well understood, since research in problem solving is substantial. The results of that research are not commonly used in textbooks and other teaching materials.

The difficulty students have in learning to solve problems limits the effectiveness of problem solving as a tool in exploring, using, and thoroughly understanding the concepts of their subject matter. These solution presentations address several of those difficulties.

The most important idea illustrated here is the development of problem solutions through using concepts of the subject matter. This premise means starting the problem solution by explicitly stating the law, definition or principle that directly responds to the question asked.

Such a beginning is in contrast to the typical textbook presentation in which the solution proceeds as if the student has already mastered the subject and developed the sophistication of the expert. In textbooks the guiding principle being used in solving the problem makes its first appearance at the end. Thus the first marker the expert made in solving the problem is the last to appear in the presentation of the solution. At that point various numerical values that have been obtained in the early part of the solution are substituted and the problem is considered solved. Some observations on this have been reported by Dall'Al-ba et al (1993).

After the initial statement of the law, definition or principle that will solve the problem, the solutions attached here proceed in a logical, step-by-step manner with successive steps being guided by cues derived from the current state of the solution. This is similar to computer methods that apply artificial intelligence to problem solving. (Soar is an example. See Newell (1990); also Newell and Simon (1972).)

The problem solution continues to evolve in a cues - to - pattern - to - cues - to - pattern process such as described by Margolis (1987). The solution emphasizes using the concepts to provide the production rules to move from one state of the solution to another. In the typical problem these production rules make use of additional physics or mathematical concepts.

The presentations provided here show the formal reasoning that occurs in problem solving. They do not, of course, show the internal reasoning. They provide an environment in which the problem and associated concepts can be explored and internal reasoning promoted.

The problem solutions illustrated in this presentation make strong use of verbalization. The value of this has been demonstrated by Whimbey (1984) with TAPS (Think Aloud Pair Solving), and by others. It is well known that students are quite capable of solving problems without understanding the concepts. (Halloun & Hestenes (1987)). Verbalization is a necessary part of coming to understand concepts. Formally including this in the solution organization, as shown in the examples, provides an avenue for exercising verbalization.

The verbal statements are similar to the explanations a person would make when explaining a step to someone else. In that way understanding the ideas (or lack of that understanding) comes strongly into play. This communication between the problem solver and his "self" is actually essential but is slighted in the teaching/learning process.

In the problem solutions shown the first statement is in essence the answer to the question. It provides a means of starting a solution: simply respond to what the problem asks for with no attention paid to the particulars of the problem. This response then asks for additional information.

The solution evolves by recursively applying this process. It follows the selective encoding and selective combination described by Sternberg (1984). This reinforces and further defines the four step problem solving process described by Polya.

There may well be several appropriate starting points for the solution of a problem. Solving a problem by starting with the various appropriate concepts is valuable to development of problem-solving and thinking skills as well as to development of an understanding of the concepts employed and their interaction.

Different starting points generate different paths through problem space and draw on different elements of knowledge space. Solving a problem using various solution paths provides a means of exploring the problem. This process leads to understanding the problem and the ideas that support its solution.

A problem solution consists of a set of nested solutions of subproblems, shown in the presentations by indenting the subproblems. This logical organization is widely used in such common things as outlines, tables of contents, organizational charts, and computer programming.

Recognizing the existence of subproblems addresses the situation described by Staver (1986), in which students have increasing difficulty with problems as the number of independent variables and conditions is increased. The resulting overload of the working memory defeats both the solution of the problem and the development of an appreciation for the concepts that are involved.

Making use of the fact that problem solving has a grammar, and that this grammar leads naturally to an indented structure, does much to alleviate this overload. Students can solve simple problems involving only one variable or condition. More complex problems are simple problems combined in an interacting, dynamic manner.

It is therefore possible to handle a complex problem because each subproblem is a simple problem. Subproblems themselves may lead to additional sub-subproblems. Solving any level of subproblem supplies results to the superior level.

The process is quite similar to functions in a computer program which supply results to the calling function and so on up to the main program. The subproblem is not involved in knowing the whole problem. It needs only to supply results to the calling problem.

Although problem solving has long been widely assumed to be an important part of the transfer of knowledge, the validity of this assumption has little research basis other than the striking results in science and technology that have resulted from the accomplishments of people trained under that assumption.

Some question about the effectiveness of good problem solving procedure has been pointed out by Heller and Reif (1984) and Costa (1993).

Inadequate problem-solving skills are amplified by the existence of sketchy examples in textbooks. Sound problem-solving principles are not widely available.

Students are exposed to fragmentary problem solving. The fragmented, inverted solutions in textbooks and answer books are the models presented to them. The opportunity to make contact with the much broader schema that the expert actually uses is not made available to students. Indeed even a casual examination of textbooks shows readily identifiable instances in which deliberate efforts have been made to *protect* the student from being confronted with a broader schema.

The student just does not know what steps to take when confronted with a problem, or is confused by conflicting examples. That this should be the case when such a substantial understanding of problem solving actually exists is unfortunate.

Failing to make use of information available from research in problem solving is a deterrent to the negotiation of knowledge transfer that is meant to occur in the educational process. Reif (1995) discusses these matters quite thoroughly.

Physics has been said to be "common sense represented in mathematical terms." With the problem solving presentations shown here we can bring common sense into mathematical representations for physics and other subjects.

References

Victoria Brookhart Costa, "School Science as a Rite of Passage; A New Frame for Familiar Problems," J. of Research in Science Teaching, 30, (7), 649-668 (1993). Return to text.

Gloria Dall'Alba, et al., "Textbook Treatment of Students' Understanding of Acceleration," J. of Research in Science Teaching, 30, (7), 621- 635 (1993). Return to text.

T. L. Heller and F. Reif, "Prescribing Effective Human Problem-Solving Processes: Problem Solving in Physics Cognition and Instruction," Cognition and Instruction, Vol. 1, No. 2, p. 177, 1984. Return to text.

Ibrahim A.Halloun and David Hestenes, "Common Sense Concepts about Motion," Am. J. Phys., 53, 1056 (1987). Return to text.

Howard Margolis, *Patterns, Thinking, and Cognition* (The University of Chicago Press, Chicago, 1987). Return to text

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Allen Newell, *Unified Theories of Cognition* (Harvard University Press, 1990). Return to text.

Allen Newell and Herbert A. Simon, *Human Problem Solving* (Prentice Hall, Englewood Cliffs, NJ, 1972). Return to text.

Frederick Reif, "Millikan Lecture 1994: Understanding and teaching important scientific thought processes," American Journal of Physics, 63, (1), 17-32 (1995). Return to text.

J. R. Staver, "The effects of problem format, number of independent variables, and their interaction on student performance on a control of variable reasoning problem," J. of Research in Science Teaching, 23, (6), 533-542 (1986). Return to text.

Robert Sternberg, *Beyond IQ* (Cambridge University Press, 1984). Return to text.

Gyargy Polya, *How to solve it* , (Princeton University Press, 1945). Return to text.

Arthur Whimbey, "Think Aloud Pair Solving -TAPS; The Key to Higher Order Thinking in Precise Processing," Educational Leadership, 42 (1), 66-70. (1987) Return to text.